



**Handbook on Theoretical
Pricing of Assets Eligible as
Collateral in the Markets to
which CCP Service is provided**

Central Counterparty Department

June, 2016

TABLE OF CONTENTS

| | | |
|------|---|----|
| I. | INTRODUCTION..... | 3 |
| II. | THEORETICAL PRICING MODELS | 4 |
| A. | Theoretical Pricing of Bonds and Bills..... | 4 |
| a) | Stripping cash flows | 4 |
| b) | Constructing yield curve | 5 |
| i. | Linear Spline Method | 5 |
| ii. | Cubic Spline Method | 6 |
| iii. | Bootstrap Method | 7 |
| B. | Examples for Bonds and Bills Theoretical Pricing..... | 8 |
| a) | Discounted Security Theoretical Pricing Example | 8 |
| b) | Fixed Coupon Rate Security Theoretical Pricing Example | 9 |
| c) | Floating Coupon Rate Security Theoretical Pricing Example | 11 |
| d) | CPI-Indexed Government Bond Theoretical Pricing Example | 12 |
| C. | Equity and Investment Fund Theoretical Pricing | 14 |

I. INTRODUCTION

The most important function of the institutions providing central counterparty (CCP) service is that they appear before each participant as a single counterparty by assuming the obligations of the parties against each other upon matching of the buying/selling transactions of the market participants and thus, enable centralization of the risks in a structure with a greater capacity to cover and manage the risks.

The main instrument used by the central counterparty institutions in providing this service is the risk and collateral management. In order for the risk and collateral relationship to be monitored with respect to the CCP-service provider institutions, the collateral obligations and margin values must be calculated in an accurate manner.

On the other hand, performing the margin valuation in a precise manner is dependent on the extent of determining the values of collateral in a manner to reflect the market conditions and eliminating the market risk that collateral faces. The key element in the margin valuation process for the CCP institutions which deduct the possibility of any price change in the non-cash assets accepted as collateral from the market value of such asset to eliminate market risk is that the values of non-cash assets whose market value has not yet been formed on the day the valuation is conducted must particularly reflect the market conditions.

The procedures for determining the prices to be used in the valuation of assets subject to collateral, the types of assets for which theoretical price is to be used as well as the order for theoretical price use have been announced by Takasbank in the market procedures for all markets to which it provides CCP service. Accordingly, Takasbank can use the theoretical prices it would determine in case a non-traded asset is included among the assets eligible as collateral at the intraday or end-of-day calculation times. Explanations about the theoretical price model for the non-cash assets whose theoretical price model has been determined are given in the following sections.

II. THEORETICAL PRICING MODELS

A. Theoretical Pricing of Bonds and Bills

In cases when no intraday price is formed in the market up until the moment at which the valuation of bonds/bills subject to collateral is performed, Takasbank shall make a theoretical pricing for the relevant instrument. In order for the theoretical pricing of the bonds/bills to be made, the financial asset must be stripped into the cash flows by taking account of its pricing characteristics and a yield curve must be constructed for the day on which the valuation is performed.

After constructing the cash flows and the yield curve, the stripped cash flows shall be discounted at the simple interest rates generated through the linear interpolation method, by using the yield curve of the relevant day. The sum of net present values of all discounted cash flows shall create the theoretical price.

$$\text{Theoretical Price (today)} = \sum_{i=0}^n \frac{c}{(1 + (r_i * T_i))} + \frac{A}{1 + (r_i * T_i)}$$

A = Principal amount

c = Coupon amount

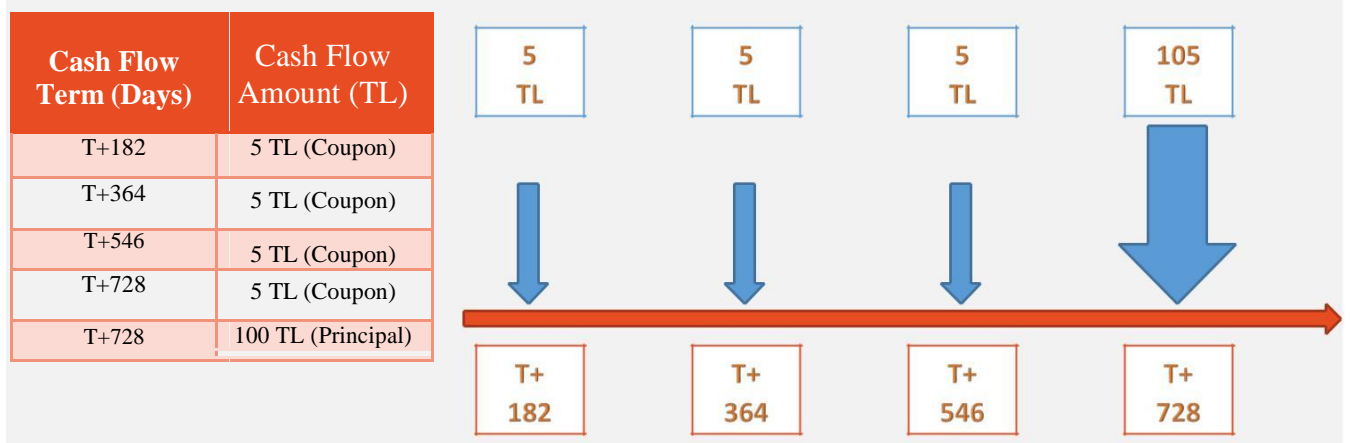
r_i = Interest

T_i = Term (Days remaining to maturity/365)

a) Stripping cash flows

In stripping the cash flows, the interest type (floating & fixed) of the financial asset, the frequency of its coupon payment, if any, and its coupon interest and maturity shall be taken into account.

For example; the cash flows on the issue date of a bond issued on T day with a nominal value of 100 TL and a term of 2 years, at 5% coupon interest with semiannual coupon payment are given below.

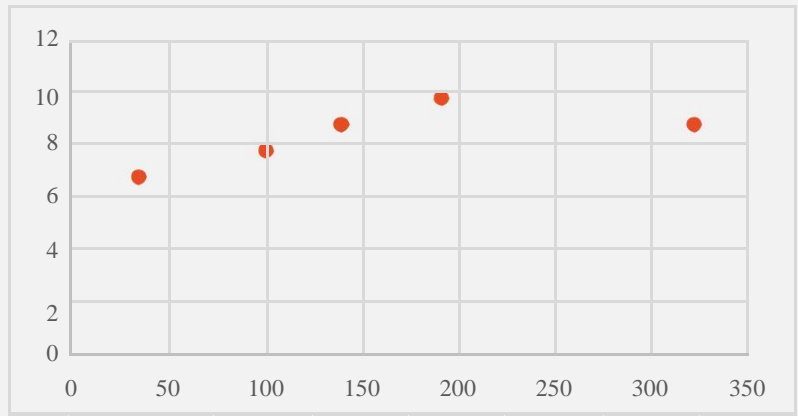


b) Constructing yield curve

A yield curve can be defined as a graphical illustration of the relationship at a specific point in time between the various maturities of an investment instrument and its yields at such maturities. In other words, a yield curve is a curve demonstrating the rates of return differing across maturities of the financial instruments having same credit risk, liquidity and tax characteristics but different maturity dates. In the yield curve it uses to determine the theoretical prices of the bonds and bills it accepts as collateral, Takasbank uses the simple interest rates of the discounted and fixed coupon rate debt instruments issued by the Republic of Turkey Undersecretariat of Treasury and traded on the relevant day. Accordingly, the theoretical prices of GDDS and of Eurobonds issued by the Republic of Turkey Undersecretariat of Treasury are set by using the risk-free interest rates.

The first step in building a Turkish Lira yield curve is to place the observed interest rates of the discounted Government Domestic Debt Securities on the maturity/yield (interest rate) coordinate axis.

| ISIN | Term to Maturity (Days) | Interest Rate (%) |
|--------|-------------------------|-------------------|
| TRTAAA | 35 | 8 |
| TRTBBB | 101 | 9 |
| TRTCCC | 140 | 10 |
| TRTDDD | 192 | 11 |
| TRTEEE | 323 | 10 |



The next step in building a yield curve is to combine the observed interest rates and transform them in the maturity-yield space from discrete pattern to continuous form. Takasbank theoretical price module can combine the observed interest rate points by the linear and cubic spline methods. The explanations about the said methods are given below.

i. Linear Spline Method

If the linear spline method is used in the process of transforming the interest rates from discrete pattern to continuous pattern to build a yield curve, the observed interest rate points are combined with a first degree function. The mathematical function that must be used for the generated linear interest interpolation is given below.

$$f(r_x) = r_i + m_{i,i+1} * \left(\frac{T_x - T_i}{365}\right)$$

r_x = Interest rate to be calculated for point x.

r_i =i Interest rate known for point i.

$m_{i,i+1}$ =Slope between the interest rate known for point i and the interest rate known for point (i+1).

T_x = Number of days remaining to maturity at point x ($T_x \in [T_i, T_{i+1}]$)

T_i = Number of days remaining to maturity at point i.

The yield curve is made continuous by creating a new function between every 2 points whose interest rates are known. As the number of observed interest rates used for designing a yield curve increases, it is assumed that the yield curve's ability to explain the maturity-yield preferences also increases.

ii. Cubic Spline Method

The main purpose of the cubic spline method which generally produces precise results in pricing of the financial instruments with nonlinear nature is to create a yield curve whose 1st derivative and 2nd derivate are flatter. Thus, the resulting inter-point function is 3rd degree. The function between 2 points can be shown as follows:

$$S_j(t) = a_j + b_j * \frac{(t - t_j)}{365} + c_j * \left(\frac{t - t_j}{365}\right)^2 + d_j * \left(\frac{t - t_j}{365}\right)^3$$

$S_j(t)$ = Interest rate to be calculated for point j.

t = Number of days remaining to maturity ($t \in [t_j, t_{j+1}]$)

t_j = Number of days remaining to maturity at time j.

In order for the yield curve to become continuous, a_j , b_j , c_j and d_j values between every 2 points whose interest rates are known must be found. In order for these four unknowns to be found, 4 polynomials whose mathematical result is known are required:

- Since it is the initial interest rate data between 2 points; $S_j(x_j) = y_j$
- Since it is the final interest rate data between 2 points; $S_j(x_{j+1}) = y_{j+1}$
- Since the 1st derivative must be equal to zero; $S'_j(x_j) = 0$
- Since the 2nd derivative must also be equal to zero; $S''_j(x_j) = 0$

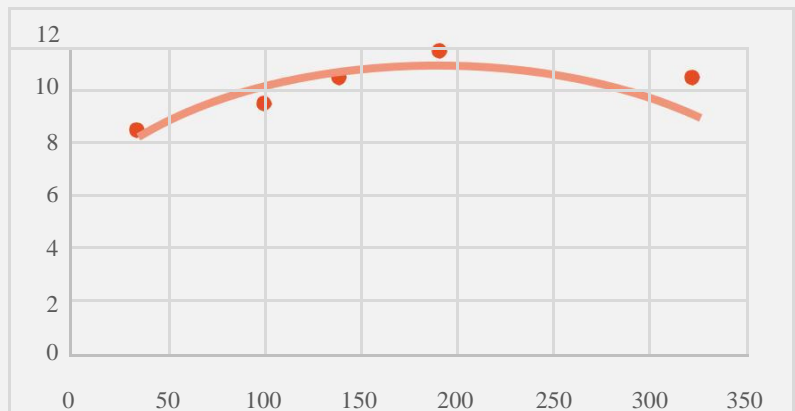
a_j , b_j , c_j and d_j haircuts are found for each range as a result of the calculations made by the outcome of these known polynomials and the yield curve is made continuous.

iii. Bootstrap method

The third step in building a yield curve is to add the fixed coupon rate bonds to the yield curve and extend the maturity period covered by the curve. For that purpose, the cash flows of the fixed coupon rate bonds that have been traded, in other words, have a certain market price and to be used in building the yield curve are firstly stripped starting from the one with the shortest term. The net present values of the cash flows falling into the maturity period of the current yield curve are calculated by interpolating the interests in the yield curve, and an interest rate iteration is made for the cash flows falling out of the current yield curve's maturity period, whose interest rates are not clear; and the main purpose of such interest iteration is to determine the interest rate that equates the sum of all coupon cash flows with the market price of the bond.

For example; let's assume that the yield curve generated by placing the interest rates of the discounted Government Domestic Debt Securities on the maturity-yield (interest rate) coordinate axis and filling out their lags is as given in the below figure. The maturity period of the yield curve generated through the discounted GDDS is 323 days.

| ISIN | Term to Maturity (Days) | Interest Rate (%) |
|--------|-------------------------|-------------------|
| TRTAAA | 35 | 8 |
| TRTBBB | 101 | 9 |
| TRTCCC | 140 | 10 |
| TRTDDD | 192 | 11 |
| TRTEEE | 323 | 10 |



The cash flows of the fixed rate bond whose price is known (99 TL) and to be used for extending the yield curve are given below.

| Coupon Rate | Remaining Term to Coupon (Days) | Known Rate of Returns | Net Present Value |
|-----------------------------|---------------------------------|-----------------------|-------------------|
| 5 | 170 | 10.5 | 4.767 |
| 105 (principal included) | 350 | X | Y |

Given the fact that the first cash flow of the bond falls into the maturity period of the curve, the net present value of that cash flow can be calculated by way of interpolating the interest rate corresponding to 170 days from the yield curve (4.767 TL). Hence, the value of the cash flow whose interest rate falls out of the curve's maturity can be found by using the bond's market value and the value of the cash flow whose interest is known (99 TL- 4.767 TL = 94.233 TL). By using such value, the simple interest rate for 350 days is calculated through the following formula.

$$94,233 \text{ TL} = \frac{105}{1 + r_{(350)} * \frac{350}{365}}$$

$$r_{(350)} = 11,92\%$$

This process referred to as Bootstrap is repeated for all assets including the longest term bond whose price has been formed in the market. The following section includes examples on theoretical price calculation on the basis of bond and bill types.

B. Examples for Bonds and Bills Theoretical Pricing

a) Discounted Security Theoretical Pricing Example

In calculating the theoretical price of discounted securities, the following process steps are followed:

- i. The discounted security is stripped into its cash flows (The only cash flow of a discounted security occurs at its maturity and that amount equals to its nominal value).
- ii. The interest rate corresponding to the maturity date is interpolated from the current yield curve (In the interest rate interpolation, the function used to construct the yield curve is applied. For example, if the yield curve is constructed by the linear spline method, the linear spline function shall be applied in the interest rate interpolation).

- iii. The cash flow is discounted by the derived interest rate (The following equation shall be used to calculate the net present value of the cash flow).

$$\text{Discounted Government Bond Theoretical Price (today)} = \frac{A}{1 + (r*T)}$$

Discounted Government Bond Theoretical Price (today)
= Theoretical price on the day of calculation

A = Principal amount
 r = Simple interest rate of the maturity in the yield curve
 T = Term of the bond (*Tenor*)

Discounted Security Pricing Example:

| | | | | |
|-------------------------|------------------------|--------------------------------|--------------------------|---|
| Sample Bond | TRT030816TXX | | | |
| <i>Calculation Date</i> | <i>Redemption Date</i> | <i>Term to Maturity (Year)</i> | <i>Principal Payment</i> | <i>Return for the relevant tenor in the yield curve</i> |
| 05/05/2016 | 03/08/2016 | 0,25 | 100 | 9,5% |

$$\text{Theoretical price of discounted government bond on the relevant day} = \frac{100}{1 + (0,095 * 0,25)}$$

Theoretical price of discounted government bond on the relevant day = 97,68 TL

The theoretical price of discounted government bond in the example is 97,68 TL.

b) Fixed Coupon Rate Security Theoretical Pricing Example

In calculating the theoretical price of fixed coupon rate securities, the following process steps are followed:

- i. The fixed coupon rate security is stripped into its cash flows (When stripping the fixed coupon rate securities into their cash flows, a cash flow up to the coupon amount occurs at the coupon periods and up to the principal amount (nominal amount) at the maturity date).
- ii. The interest rates of all cash flows corresponding to the maturity dates are interpolated from the current yield curve (In the interest rate interpolation, the function used to construct the yield curve is applied. For example, if the yield curve is constructed by the

linear spline method, the linear spline function shall be applied in the interest rate interpolation).

- iii. The cash flows are discounted by the derived interest rates.
- iv. The net present values of the calculated cash flows are summed-up and the theoretical price shall be obtained by this way.

$$\text{Fixed Rate Bond Theoretical Price}_{(\text{today})} = \sum_{i=0}^n \left(\frac{c}{1 + (r_i * T_i)} + \frac{A}{1 + (r_i * T_i)} \right)$$

Fixed Rate Bond Theoretical Price (today) = Theoretical price on the day of calculation

c = Fixed coupon amount

A = Principal amount

r_i = Simple interest rate of the relevant maturity in the yield curve

T_i = Term of the relevant cash flow (Tenor)

n = Number of cash flows

Fixed Rate Government Bond Theoretical Pricing Example:

| Sample Fixed Rate Bond | | TRT020817TXX | | | |
|-------------------------|-------------|-------------------------|-------------------|--|---|
| Periodic Coupon Payment | | 3 TL | | | |
| Calculation Date | Coupon Date | Term to Maturity (Year) | Amount to be Paid | Return for the relevant tenor in the yield curve | Net Present Values of Cash Flows |
| 05/05/16 | 03/08/16 | 0,25 | 3 | 9,25% | $\frac{3}{1 + (0,0925 * 0,25)} = 2,93$ |
| 05/05/16 | 01/02/17 | 0,76 | 3 | 9,5% | $\frac{3}{1 + (0,095 * 0,76)} = 2,80$ |
| 05/05/16 | 02/08/17 | 1,26 | 103 | 9,75% | $\frac{103}{1 + (0,0975 * 1,26)} = 91,73$ |

Theoretical price of fixed rate government bond on the relevant day = 2,93 + 2,80 + 91,73 = 97,46

Theoretical price of fixed rate government bond in the example is 97,46 TL.

c) Floating Coupon Rate Security Theoretical Pricing Example

In calculating the theoretical price of floating coupon rate securities, the following process steps are followed:

- i. The floating coupon rate security is stripped into its cash flows (When stripping the floating rate securities into their cash flows, it is assumed that the known coupon rate shall be applicable for all coupon payments; and upon splitting the floating coupon rate securities into their cash flows accordingly, a cash flow at an amount equal to the last known coupon amount occurs at the coupon periods and up to the principal amount (nominal amount) at the maturity date).
- ii. The interest rates of all cash flows corresponding to the maturity dates are interpolated from the current yield curve (In the interest rate interpolation, the function used to construct the yield curve is applied. For example, if the yield curve is constructed by the linear spline method, the linear spline function shall be applied in the interest rate interpolation).
- iii. The cash flows are discounted by the derived interest rates.
- iv. The net present values of the calculated cash flows are summed-up.

$$\text{Floating Coupon Rate Bond Theoretical Price}_{(\text{today})} = \sum_{i=0}^n \left(\frac{c}{1 + (r_i * T_i)} + \frac{A}{1 + (r_i * T_i)} \right)$$

Floating Coupon Rate Bond Theoretical Price (today) = Theoretical price on the day of calculation

c = Coupon amount to be paid in the next period

A = Principal amount

r_i = Rate of return of the relevant maturity in the yield curve

T_i = Term of the relevant cash flow (By year)

n = Number of cash flows

Floating Coupon Rate Government Bond Theoretical Pricing Example:

| Floating Coupon Rate Bond | | | | TRT020817TXX | |
|---------------------------|--------------------|--------------------------------|--------------------------|---|---|
| Last Known Coupon Rate | | | | 4 TL | |
| <i>Calculation Date</i> | <i>Coupon Date</i> | <i>Term to Maturity (Year)</i> | <i>Amount to be Paid</i> | <i>Return for the relevant tenor in the yield curve</i> | <i>Net Present Values of Cash Flows</i> |
| 05/05/2016 | 03/08/2016 | 0,25 | 4 | 9,25% | $\frac{4}{[1 + (0,0925 * 0,25)]} = 3,91$ |
| 05/05/2016 | 01/02/2017 | 0,76 | 4 | 9,5% | $\frac{4}{[1 + (0,095 * 0,76)]} = 3,73$ |
| 05/05/2016 | 02/08/2017 | 1,26 | 104 | 9,75% | $\frac{104}{[1 + (0,0975 * 1,26)]} = 92,62$ |

Theoretical price of floating coupon rate bond on the relevant day = 3,91 + 3,73 + 92,62 = 100,26

Theoretical price of floating coupon rate government bond is 100,26 TL.

d) CPI-Indexed Government Bond Theoretical Pricing Example¹

In calculating the theoretical price of CPI-indexed securities, the following process steps are followed:

- i. The CPI-indexed coupon security is stripped into its cash flows (When stripping the security into its cash flows, it is assumed that a fixed return up to the risk premium would occur at each coupon payment; and upon splitting the CPI-indexed securities into their cash flows accordingly, a cash flow at an amount equal to the risk premium occurs at the coupon periods and up to the principal amount (nominal amount) at the maturity date).
- ii. The interest rates of all cash flows corresponding to the maturity dates are interpolated from the current yield curve. (In the interest rate interpolation, the function used to

¹ Theoretical pricing shall be made as of 2018.

construct the yield curve is applied. For example, if the yield curve is constructed by the linear spline method, the linear spline function shall be applied in the interest rate interpolation).

- iii. The cash flows are discounted by the derived interest rates.
- iv. For the coupon periods, an index growth forecast is made (In forecasting the index growth, the CPI's future values corresponding to the coupon periods are first forecasted, and then the index growth rate is calculated by way of dividing the values forecasted for the future to the value of the index on the issue date).
- v. Each discounted cash flow is scaled by multiplying it by the index growth rate corresponding to the relevant term.
- vi. The theoretical price is calculated by summing up the scaled cash flows.

$$\text{Theoretical Price}_{(\text{today})} = \sum_{i=0}^n \left(\frac{A_i}{1 + (r_i + T_i)} * \frac{\text{Index}_i}{\text{Index}_{\text{issue}}} \right)$$

Theoretical Price _(today) = Theoretical price on the day of calculation

A_i = Coupon amount in the relevant cash flow

r_i = Rate of return of the relevant maturity in the yield curve

T_i = Term of the relevant cash flow (By year)

Index_i = Benchmark index of the relevant cash flow calculated by Takasbank based on the benchmark indices published by the Undersecretariat of Treasury.

$\text{Index}_{\text{issue}}$ = Benchmark index published by the Undersecretariat of Treasury on the bond's issue date.

n = Number of cash flows

CPI-Indexed Government Bond Theoretical Pricing Example:

| | |
|--|--------------|
| Sample CPI-Indexed Bond | TRT020817TXX |
| Periodic Coupon Payment | 1,5 TL |
| Benchmark index on the bond's issue date | 283000 |

| Calculation Date | Coupon Date | Term to Maturity (Year) | Amount to be Paid | Simple Interest | Benchmark Index | Periodic Net Present Values |
|------------------|-------------|-------------------------|-------------------|-----------------|-----------------|---|
| 05/05/16 | 03/08/16 | 0,25 | 1,5 | 9,25% | 284000 | $\frac{284000}{1 + (0,0925 * 0,25)} + \frac{1,5 * 283000}{1 + (0,0925 * 0,25)} = 1,47$ |
| 05/05/16 | 01/02/17 | 0,76 | 1,5 | 9,5% | 285000 | $\frac{285000}{1 + (0,095 * 0,76)} + \frac{1,5 * 283000}{1 + (0,095 * 0,76)} = 1,41$ |
| 05/05/16 | 02/08/17 | 1,26 | 101,5 | 9,75% | 286000 | $\frac{286000}{1 + (0,0975 * 1,26)} + \frac{101,5 * 283000}{1 + (0,0975 * 1,26)} = 91,35$ |

Theoretical price of CPI-indexed government bond on the relevant day = 1,47 + 1,41 + 91,35 = 94,23

Theoretical price of CPI-indexed government bond in the example is 94,23 TL.

C. Equity and Investment Fund Theoretical Pricing

In the theoretical pricing of the equities and exchange investment funds among the assets subject to collateral, the theoretical price shall be modelled by assuming that the equity or the exchange investment fund whose price has not been formed follow a parallel trend to the index to which they belong.

The theoretical pricing formula used by Takasbank is as follows:

$$Price_t = \left[\frac{BIST100 Index - BIST100 Index_{t-1}}{BIST100 Index_{t-1}} + 1 \right] * Price_{t-1}$$

Equity Theoretical Pricing Example:

| | |
|--|-----------------|
| <i>Previous day's price of equity (t - 1)</i> | <i>12.15 TL</i> |
| <i>Relevant day's value of BIST100 Index (t)</i> | <i>86378.33</i> |
| <i>Previous day's value of BIST100 Index (t - 1)</i> | <i>85260.85</i> |

$$\text{Price of equity on the relevant day (t)} = \frac{[86378.33 - 85260.85 + 1]}{85260.85} * 12.15$$

$$\text{Price of equity on the relevant day (t)} = 101.311\% * 12.15 = 12.31 \text{ TL}$$

Theoretical price of equity in the example is 12.31 TL.

Contact details for any further questions:

Central Counterparty Department

N. Burak AKAN: + (90) 212 315 22 43

A.Cüneyt MEHMETOĞLU: + (90) 212 315 23 29

Cihangir Şahin: + (90) 212 315 20 36